The Geometry of Change

In his famous letter to Huyghens concerning his discovery of the significance of the square roots of negative numbers, G.W. Leibniz stated clearly his recognition that this investigation originated with the scientists of ancient Greece: "There is almost nothing more to be desired for the use which algebra can or will be able to have in mechanics and in practice. It is believable that this was the aim of the geometry of the ancients (at least that of Apollonius) and the purpose of loci that he had introduced . . . ."

Understanding the implication of Leibniz's statement is crucial to grasping the deeper significance of Gauss's 1799 treatment of the fundamental theorem of algebra.

Leibniz's statement will either baffle, or enrage, a modern academic, but such reactions only typify a broader social disease: the inability, as Lyndon LaRouche has repeatedly emphasized, to recognize the essential difference between human and beast. Like any disease, this one spreads through infectious agents that attack the defenses of the victim, causing the victim's own system to act as an agent for the aggressor. The cure for such conditions is to strengthen the targetted population's natural immunities, enabling it, not only to fight the disease, but to become permanently resistant to its effects. In this case, those natural immunities are the cognitive powers of the human mind. *Hence, the therapeutic effects of pedagogical exercises and Classical art.*

What Leibniz, Gauss, and their predecessors in antiquity understood, is that the essential distinction between man and animal is the capacity of the human mind to reach behind the domain of the senses and discover those unseen principles that govern the changes perceived in the physical universe. However, being unseen, those principles can only be discovered through changes (motions) within the domain of the senses, which in turn give rise to paradoxes concerning the relationship of the seen to the unseen. Consequently, it is the coupled interaction between the seen and the unseen that must be comprehended. Physical motion gives rise to the willful motion (passion) of the mind from one state to a higher one.

As Leibniz indicates, no formal system, such as algebra or Euclidean geometry, is capable of representing this characteristic of change that emerges from the interaction between the seen and the unseen.[1] Only a geometry of change, such as the pre-Euclidean "spherics" of Thales and the Pythagorean school, the geometry of motion associated with Archimedes, Eratosthenes, and Apollonius, Leibniz's infinitesimal calculus, or Gauss's concept of the complex domain, has such power.

Just as the origins of the discovery of the complex domain begin in the ancient Mediterranean cultures of Egypt and Greece, so do the roots of its adversary. The mode of attack has been to induce the false belief that the physical world which is seen, and the immaterial world which is unseen, do not interact, but are hermetically separated. This belief is typified by the mystery cults of ancient Babylonian and Persian cultures. The Eleatics (such as Parmenides and Zeno) sought to introduce this corruption into Greek culture, against Heracleitus and the Pythagoreans, by insisting that change is merely an illusion and does not exist.[2]

Socrates made mincemeat of Parmenides' Eleatic argument; so, after this, those who would today be called satanic, switched tactics, expressing the same evil intent through forms of Sophistry, such as admitting that change exists, but then arbitrarily defining change as the opposite of the Good, and defining the Good as that which does not change and is not corrupted by change.

After Plato discredited the trickery of Sophistry, Aristotle, while distancing himself formally from the Sophists, nevertheless propounded the same evil in a new guise. For example, writing in his *Nichomachean Ethics,* Aristotle said:

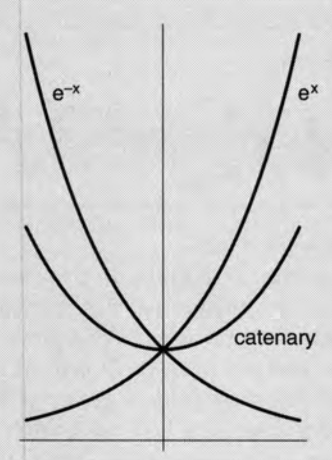


FIGURE 1. *The visible catenary is the arithmetic mean between right-handed and left-handed exponential curves.*

"This is why God always enjoys a single and simple pleasure; for there is not only an activity of movement but an activity of immobility, and pleasure is found more in rest than in movement. But change in all things is sweet, as the poet says, because of some vice; for as it is the vicious man that is changeable, so the nature that needs change is vicious; for it is not simple nor good." (*Ethics,* 1154b)

Aristotle adopted this same view toward physical motion, stating, in his *Physics,* that motion originates only from within a body, and that irregular motion, because it contains more change, is of a lesser degree than regular motion, which is itself of a lesser degree than rest.

Like the Sophists and the Eleatics, Aristotle was not developing an original argument, but reacting against Plato's repeated demonstration that the material and the immaterial are coupled:

"For, in truth, this Cosmos in its origin was generated as a compound, from the combination of Necessity and Reason. And inasmuch as Reason was controlling Necessity by persuading her to conduct to the best end the most part of the things coming into existence, thus and thereby it came about, through Necessity yielding to intelligent persuasion, that this Universe of ours was being in this wise constructed at the beginning." (*Timaeus,* 48a)

And it is the power to gain knowledge of the universe through the interaction of the seen with the unseen, the temporal with the eternal, that is human nature. Change is a characteristic, not of viciousness and vice, but of perfection:

"But as it is, the vision of day and night and of months and circling years has created the art of number and has given us not only the notion of Time but also means of research into the nature of the Universe. From these we have procured Philosophy in all its range, than which no greater book ever has come or will come, by divine bestowal, unto the race of mortals. . . . God devised and bestowed upon us vision to the end that we might behold the revolutions of Reason in the Heaven and use them for the revolvings of the reasoning that is within us, these being akin to those, the perturbable to the imperturbable; and that, through learning and sharing in calculations which are correct by their nature, by imitation of the absolutely unvarying revolutions of the God we might stabilize the variable revolutions in ourselves.

"Concerning sound also and hearing, once more we make the same declaration . . . ; music too, in so far as it uses audible sound, was bestowed for the sake of harmony. And harmony, which has motions akin to the revolutions of the Soul within us, was given by the Muses to him who makes intelligent use of the Muses, not as an aid to irrational pleasure, as is now supposed, but as an auxiliary to the inner revolution of the Soul, when it has lost its harmony, to assist in restoring it to order and concord with itself. And because of the unmodulated condition, deficient in grace, which exists in most of us, Rhythm also was bestowed upon us to be our helper by the same deities and for the same ends." (*Timaeus,* 47a-e)

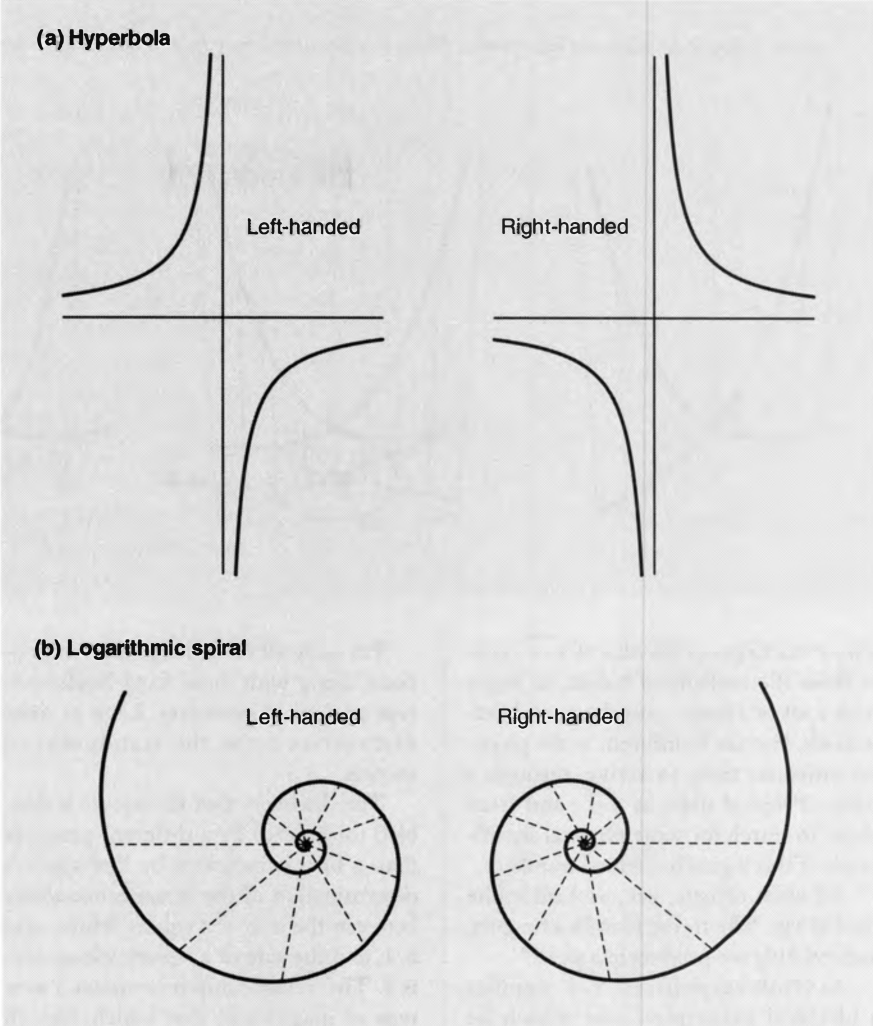


FIGURE 2. *As in the case of exponential curves, right-handed and left-handed forms of the (a) hyperbola and (b) logarithmic spiral cannot be transformed into one another within the plane of their visible existence.*

The tension of this Socratic irony, of the unchanging principles of change, is the means by which man, and the universe as a whole, perfects itself. As Kepler notes in the *New Astronomy,* it is the tension from the discovery that the planetary orbits are not circular, "that gives rise to a powerful sense of wonder which at length drives men to look into causes."

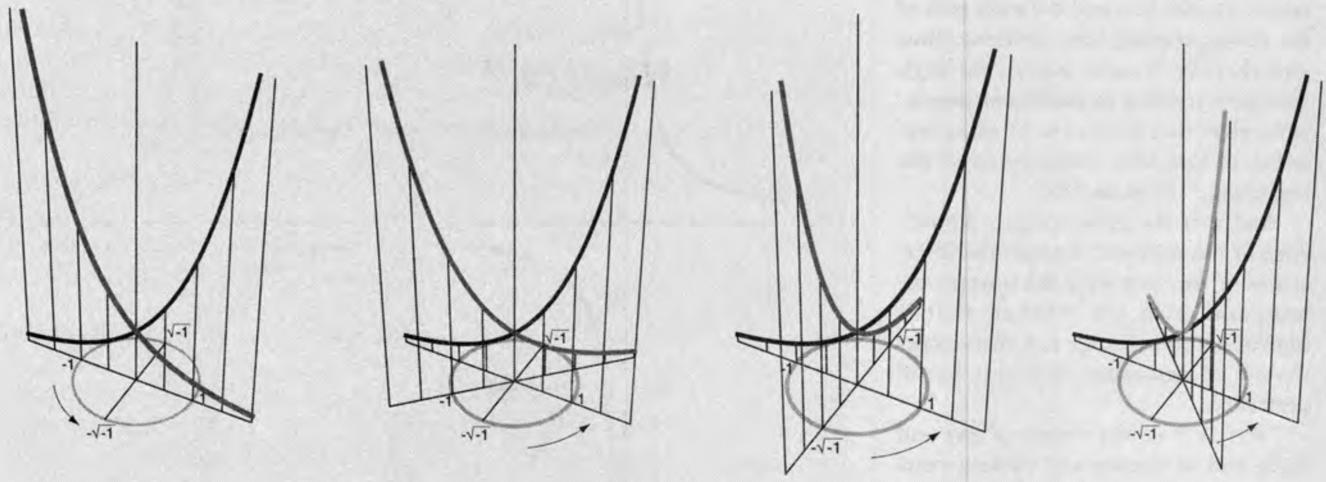
Remove that tension, as Aristotle, Euler, Lagrange, et al. do, and you excise from man his human nature, rendering him defenseless against those oligarchical forces who seek to enslave him.

$\sqrt{-1}$ and Motion

Putting aside the problem of the passionless (or, more likely, enraged) committed Aristotelean, a persistent difficulty arises for those wishing to comprehend Gauss's discovery of the complex domain. The difficulty centers on grasping the physical significance of $\sqrt{-1}$. For Euler, Lagrange, and D'Alembert, $\sqrt{-1}$ is merely a passionless definition of the solution to the equation $x^2 + 1 = 0$. All tension associated with its existence is removed by the declaration that it is a definition of something that is "impossible." "What, Me Worry about the impossible?"

The difficulty for the serious person

FIGURE 3. *Right-handed and left-handed forms can be united only by a rotational motion orthogonal to the plane of their visible existence.*



who seeks to grasp the idea of $\sqrt{-1}$, arises from the embedded habits, to begin with a set of axioms, postulates, and definitions, that are indifferent to the physical universe; then, to arrive, through a series of logical steps, at $\sqrt{-1}$; and from there, to search for some physical significance of this logically defined number.

All such efforts, are, as LaRouche used to say, "like trying to milk a he-goat, and catching the product in a sieve."

As Gauss emphasized, $\sqrt{-1}$ signifies a physical principle—one which he said, "has the deepest implications for the metaphysics of the theory of space." As a study of Gauss's early notebooks reveals, his development of the complex domain arose from the paradoxes of the "Kepler Problem" that remained unresolved by Leibniz's infinitesimal calculus.[3] Keeping that in mind, along with what was said above, the physical significance of $\sqrt{-1}$ can be demonstrated, as Leibniz indicated, by conceptualizing the unified succession of discoveries from Pythagoras through Gauss. It is only through this ironical, polyphonic approach, that insights into the physical significance of $\sqrt{-1}$ can be obtained.

This can be done quite efficiently if one has mastered the general principles expressed by the discoveries of the doubling of the square and cube, and of the catenary.

Put aside all formal algebraic conceptions, along with those fixed Euclidean-type notions of geometry. Look at these discoveries from the standpoint of motion.

The discovery that the square is doubled (or halved) by a different principle than a line, is indicated by Pythagoras's determination of the *incommensurability* between the side of a square whose area is 1, and the side of a square whose area is 2. This relationship determines a new type of magnitude, that which, like all numbers, is not susceptible to formal definition, outside the physical relationship from which it originates. In other words, $\sqrt{2}$ is not the number 1.14142135..., but a magnitude that exists only within the physical relationship of two squares whose areas are in the ratio of 1:2.

As Plato reports in the *Theaetetus,* this magnitude is only a special case of a whole class of magnitudes, that can be characterized as the relationship of one geometric mean between two extremes.

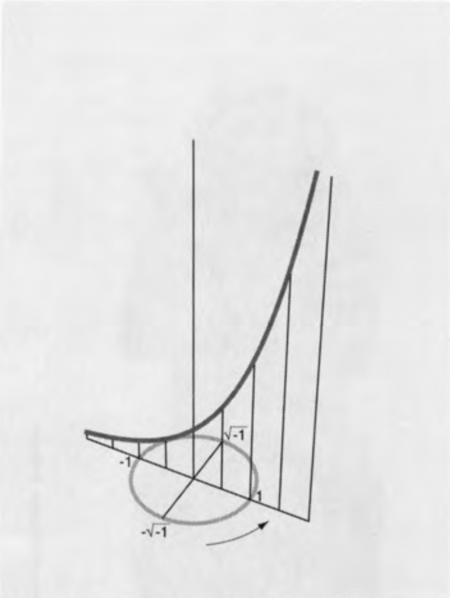
This whole class of magnitudes, however, can be generated by one type of physical motion, specifically, circular action.

However, an entirely new type of magnitude emerges when doubling the cube. As Plato stated in the *Timaeus,* if God had created the world flat, it would only be necessary to have one mean between two extremes, but God created the world solid, so it is always necessary to find two.

As Archytas's construction demonstrates, this new type of magnitude cannot be generated by simple circular action, but requires circular action acting orthogonally on circular action. This action on action is what generates the torus, cylinder, and cone of Archytas's famous construction. Subsequently, Menaechmus and Apollonius demonstrated the more general form of the principle of Archytas's construction through the development of conics. For them, as well as Archimedes, Eratosthenes, *et al.,* it was this higher type of physical action, *expressed by motion acting on motion,* that generated the relationships that are manifest in solid bodies such as squares and cubes. *Contrary to Aristotle, motion doesn't originate in bodies. Bodies originate in motion.*

To repeat: The magnitudes associated with one geometric mean between two extremes are a species of magnitudes generated by one principle of motion, i.e., circular action, and the magnitudes associated with two geometric means are a species of magnitudes generated by another class of motion, i.e., conical action.

However, as Leibniz and Bernoulli indicated, the latter type of motion (conical), actually generates a class of classes



of magnitudes. Each separate class is characterized by the number of means between two extremes and is identified with a specific type of power. (For example, the fourth power requires three geometric means between two extremes, the fifth power four geometric means, etc.)

Such magnitudes Leibniz called "algebraic," or alternatively, "algebraic powers." Magnitudes associated with the higher, class of classes, Leibniz called "transcendental." These transcendental magnitudes exist outside the domain of the algebraic. Nevertheless, the two are connected, because the higher transcendentals generate the lower algebraic. *As Leibniz states, the transcendentals are the ones that express the relationships that arise within the physical universe.*

The physical significance of the first two classes of algebraic powers, squares and cubes, is evident from the problems of Pythagoras and Archytas. What is the physical significance of the motion that generates the entire class of algebraic powers?

That significance is found in Leibniz's solution to the catenary problem. As an expression of the principle of least-action, the catenary is the form of a hanging-chain that is motionless. But, as Leibniz demonstrates, the chain's stillness reflects the motion which generates the higher transcendental magnitudes. In the case of the catenary, that motion is expressed as two exponential curves [SEE Figure 1].

The visible catenary, Leibniz shows, is the arithmetic mean between two exponential curves. But that is only half the story. To paraphrase Plato from the *Timaeus,* since God made the catenary with two exponentials, what is the nature of the mean that binds them? Or, in other words, what physical action produces two exponentials, together?

An insight can be gained by looking at the other expressions of the exponential relationship, such as the hyperbola, and the logarithmic spiral [SEE Figure 2]. In all three cases, there are two distinct forms, left-handed and right-handed. These two forms cannot be transformed one into the other within the plane of their visible existence.

But as the catenary demonstrates, the physical universe is happy only when both forms are united into one. What is the nature of the species of motion that unites both left- and right-handed exponentials? That motion is a rotation orthogonal to the visible plane of the two curves [SEE Figure 3]. (It is strongly recommended that physical models of this motion be built.)

This is the action that Gauss understood as the physical action that gives rise to $\sqrt{-1}$. To see this, look at one of the exponentials. It generates all the algebraic powers, increasing in one direction and decreasing in the other direction [SEE Figure 4]. Now, look at the other exponential. It does the same thing. But, in the direction in which one increases, the other decreases, and *vice versa.* From this standpoint the two are mutually exclusive.

Yet, the catenary binds them both. If, as Gauss did, we designate one exponential as positive and the other as negative, then the two are bound by the geometric mean between 1 and –1, or $\sqrt{-1}$.

Does $\sqrt{-1}$ physically exist? Just ask the catenary.

Can it be seen?

Yes. But, only by humans. Not by animals or Aristoteleans.

—Bruce Director

1. After Leibniz-hating ideologue Leonhard Euler, and his *protégé* Lagrange, had published their fraudulent attacks on Leibniz on this crucial principle of Leibniz's original definition of his infinitesimal calculus, Gauss's 1799 Fundamental Theorem of Algebra responded with a novel, but appropriate defense of this argument made originally by Leibniz.
2. Bertrand Russell and today's proponents of "information theory" describe themselves as being in the tradition of the Eleatics.
3. See Bruce Director, "Riemann for Anti-Dummies," Part 49, Aug. 16, 2003 (unpublished).

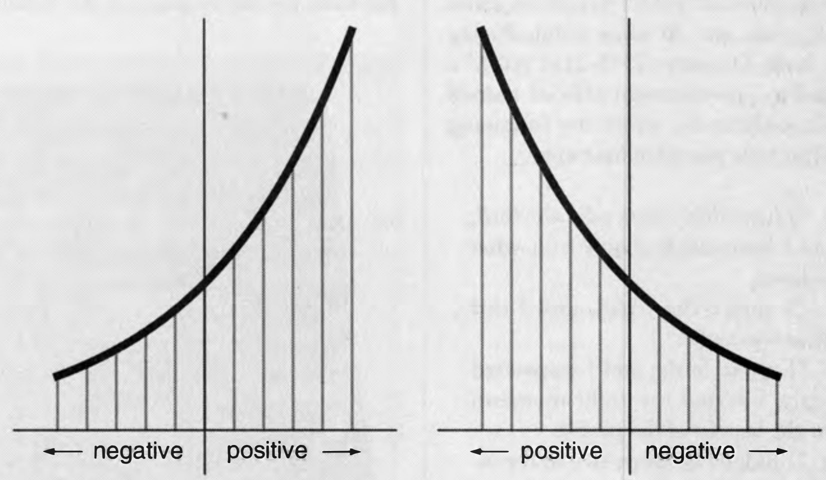


FIGURE 4. *Right-handed and left-handed exponentials generate all the algebraic powers, but in inverse directions. They are bound together by the physically existing catenary.*